UDC 004.8; 004.94 **doi:** https://doi.org/10.20998/3083-6298.2025.03.03

Oleksandr Yevtushenko[1], Oleksandr Zakovorotnyi[1]

[1] National Technical University "Kharkiv Polytechnic Institute", Kharkiv, Ukraine

RESEARCH ON THE EFFECTIVENESS OF METHODS OF THE TRAIN CRITICAL SPEED CALCULATION

Abstract. Topicality. Determining the critical velocity of railway rolling stock is a key step in the design and operation of modern trains, as exceeding this velocity leads to sustained lateral oscillations, increased lateral forces at the wheel–rail contact, and consequently a higher risk of derailment. Given the deteriorating condition of rolling stock and track infrastructure, reliable and efficient prediction of critical velocity based on modern computational methods has become increasingly necessary. **The subject of study** is the effectiveness of machine learning methods for approximating critical velocity under conditions of nonlinear dependencies and limited data availability. **The purpose of the** article is to identify the most effective method for subsequent implementation in the Driver Decision Support System (DDSS) and as a component of the train curvilinear motion model. **The following results** were obtained: MLP (Multilayer Perceptron) and GPR (Gaussian Process Regression) models demonstrated high prediction accuracy on both small and large datasets; however, MLP exhibited better scalability compared to GPR as the training dataset size increased. **Conclusion.** Based on the comparative analysis conducted, MLP is recommended as the primary model for critical velocity estimation within the DDSS and for railway transport diagnostics.

Keywords: critical velocity, rolling stock vibrations, multilayer perceptron (MLP), Gaussian process regression (GPR), support vector regression (SVR), approximation, machine learning, Driver Decision Support System (DDSS), curvilinear motion.

Introduction

Problem relevance. Research into oscillatory processes and motion instabilities of railway vehicles equipped with conventional solid-axle wheelsets is becoming increasingly relevant today, driven by the necessity to compensate for the deteriorating condition of rolling stock through comprehensive optimization of its motion.

The lowest velocity at which self-sustained lateral oscillations (hunting) of the wheelset–bogie system emerge is termed the critical velocity. Determining the critical velocity is a crucial task in the development and design of railway vehicles. If a vehicle operates at velocities exceeding the critical velocity, dynamic processes triggered by perturbations may fail to damp out and instead lead to sustained oscillations of the vehicle. Although such motions may be stable in the mathematical sense, this behavior is referred to as unstable motion. Since severe oscillations can generate high lateral forces between the wheel and rail and consequently increase the risk of track displacement or even derailment – such behavior must be avoided under normal operating conditions [1, 2]. In other words, the critical velocity imposes an upper bound on the permissible operating velocity of a railway vehicle.

Given the constant deterioration of both railway infrastructure and rolling stock, which leads to a dynamic reduction in the safe operating speed of trains in real time, the ability to quickly and reliably assess critical speed has become not just an advantage, but an operational necessity to prevent instability caused by fluctuations and reduce the risk of derailment.

Literature review. A substantial body of research has been devoted to the issue of critical velocity. Exceeding the critical velocity is identified as one of the potential causes of train derailments [3]. Studies have examined the influence of the track and its underlying substructure on the critical velocity value [4-6], as well as the possibility of determining this parameter by analyzing the effects that motion at critical velocity exerts on the track [7, 8]. Two computational approaches for critical velocity estimation are commonly discussed: the trajectory-following method, which enables automated calculation, and the brute-force method. However, due to its reliance on the periodicity of solutions, the trajectory-following method is inherently limited to strictly periodic motion patterns. Consequently, the brute-force method proves more suitable for critical velocity estimation along complex (non-periodic) trajectories [9].

Critical velocity computation is a non-trivial task. This value depends on a multitude of variables, constant parameters, and train motion characteristics many of which influence the critical velocity in a highly nonlinear manner. Given the inherent complexity of railway vehicle dynamics, direct computation of critical velocity based on the existing mathematical model of curvilinear motion dynamics [10] is impractical. Classical methods for estimating critical velocity rely on linearized stability analysis or time-domain simulation with incremental speed sweeps. However, these approaches assume idealized track geometry, neglect degradation effects (e.g. wheel profile wear, suspension hysteresis), and demand high-fidelity models that are computationally prohibitive for real-time implementation. Moreover, they often fail to account for stochastic disturbances (e.g., rail joints, track irregularities), resulting in over-optimistic estimates of stability margins.

Therefore, this study proposes to approximate the critical velocity using appropriate computational tools adapted to this specific challenge.

The purpose of the research is to comparatively evaluate three supervised regression methods – Multilayer Perceptron (MLP), Gaussian Process Regression (GPR), and Support Vector Regression

(SVR) – for their ability to approximate the critical velocity of railway rolling stock under strongly nonlinear parameter dependencies and limited data availability;

The most efficient method will subsequently be employed to develop a critical velocity estimation module within the Driver Decision Support System (DDSS) and as a component of the train curvilinear motion model.

1. Methods of critical speed calculation

Let's look at a train going through a curved section of track with a constant radius of curvature $R = 500$. Within the existing model of curved train movement, the following dependencies apply to the critical velocity V_{cr}: dependence on the conicity of the wheel pair γ, on the nominal rolling radius r_0, on the angular stiffness of the bogie-body connections C. These dependencies are nonlinear and can be conditionally represented as follows:

$$Vcr \propto \frac{r_0 C}{\sqrt{\gamma}}. \quad (1)$$

From the above relationship, it follows that the critical velocity is directly proportional to the nominal rolling radius r_0 and the angular stiffness C, and inversely proportional to the square root of the wheelset conicity γ. Based on these dependencies, approximation of the critical velocity is feasible.

The following tools suitable for critical velocity approximation are considered:

Multilayer Perceptron (MLP) – a class of feedforward artificial neural networks consisting of at least three layers: input, one or more hidden layers, and output. Except for the input layer, all neurons employ a nonlinear activation function.

Gaussian Process Regression (GPR) – a powerful and flexible non-parametric regression technique used in machine learning and statistics. It is especially useful for problems involving continuous data where the relationship between input variables and the output is unknown or highly complex. GPR is a Bayesian approach that quantifies uncertainty in predictions, making it valuable for applications such as optimization and time-series forecasting. It is based on the concept of a Gaussian process – a collection of random variables, any finite subset of which follows a joint Gaussian distribution.

Support Vector Regression (SVR) – a variant of Support Vector Machines (SVMs) used for regression tasks. Its objective is to find a function that best predicts a continuous output value for a given input. SVR can employ either linear or nonlinear kernels. A linear kernel computes the simple dot product of two input vectors, whereas a nonlinear kernel (e.g., radial basis function – RBF) captures more complex patterns in the data. The choice of kernel depends on data characteristics and problem complexity.

2. Experimental analysis of selected methods

To evaluate the performance of the selected methods in predicting critical velocity, the following standard metrics were used:

$RMSE$ – Root Mean Square Error

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(y_i^{pred} - y_i^{true}\right)^2}, \quad (2)$$

where: y_i^{true} – true (reference) critical velocity from the dataset; y_i^{pred} – predicted critical velocity; N – number of samples in the test set; R^2 – Coefficient of Determination:

$$R^2 = 1 - \frac{\sum_{i=1}^{N}\left(y_i^{true} - y_i^{pred}\right)^2}{\sum_{i=1}^{N}\left(y_i^{true} - \bar{y}^{true}\right)^2}, \quad (3)$$

where $\bar{y}^{true}$ is the mean of the true values.

Two datasets were compiled for the analysis: a small-scale and a large-scale dataset, containing variations of critical velocity corresponding to different parameter combinations (y, r_0, C). These datasets were used to train the models, all implemented in MATLAB.

The resulting prediction accuracy graphs are presented in Figures 1–6.

Figures 1–3 show the prediction accuracy of the models trained on the small dataset. It can be observed that both MLP and GPR successfully approximate the critical velocity, with GPR achieving higher accuracy than MLP. In contrast, SVR performs poorly on this task.

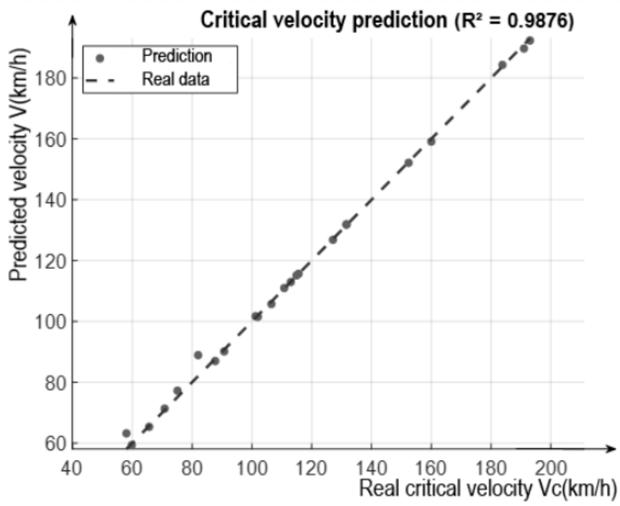

Fig. 1. Critical velocity prediction using the MLP model trained on a small dataset

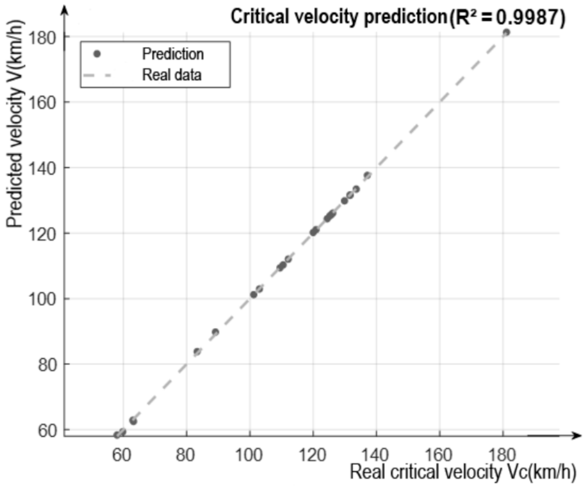

Fig. 2. Critical velocity prediction using the GPR model trained on a small dataset

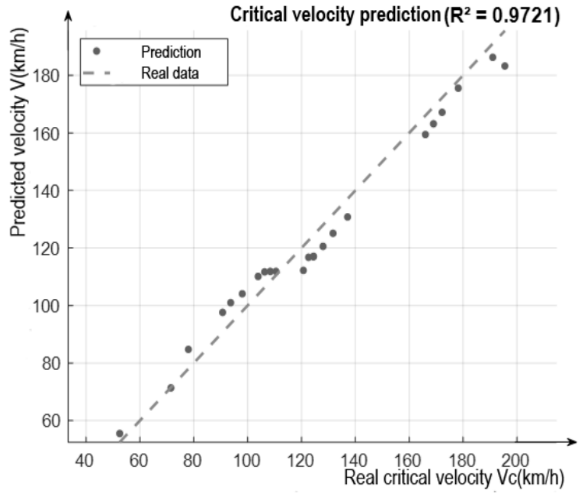

Fig. 3. Critical velocity prediction using the SVR model trained on a small dataset

It can be observed that both MLP and GPR successfully approximate the critical velocity, with GPR achieving higher accuracy than MLP. In contrast, SVR performs poorly on this task.

Figures 4–6 present results for models trained on the large dataset:

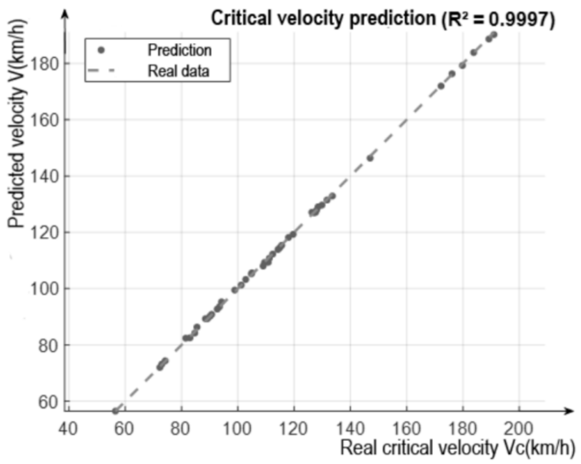

Fig. 4. Critical velocity prediction using the MLP model trained on a large dataset

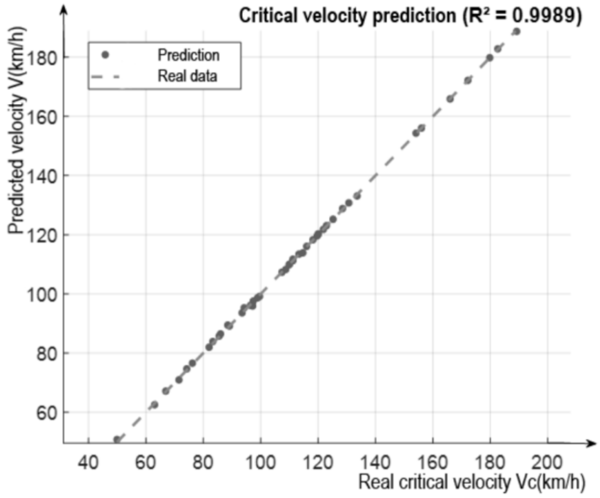

Fig. 5. Critical velocity prediction using the GPR model trained on a large dataset

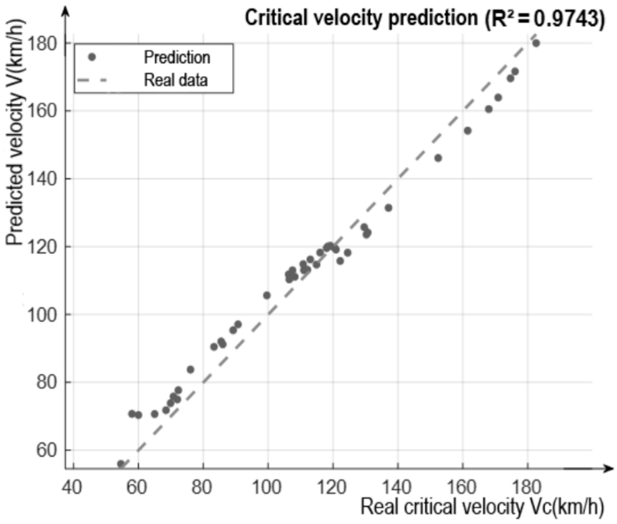

Fig. 6. Critical velocity prediction using the SVR model trained on a large dataset

On the large dataset, both MLP and GPR remain effective, but GPR accuracy decreases with increasing data volume, whereas MLP accuracy improves, demonstrating better scalability. SVR continues to underperform on both small and large datasets. Consequently, considering the prospect of further data and parameter expansion, the MLP model is the most appropriate choice for critical velocity prediction.

To further validate MLP's effectiveness, an additional comparison was performed for the dependence of critical velocity on wheelset conicity γ (with other parameters held constant):

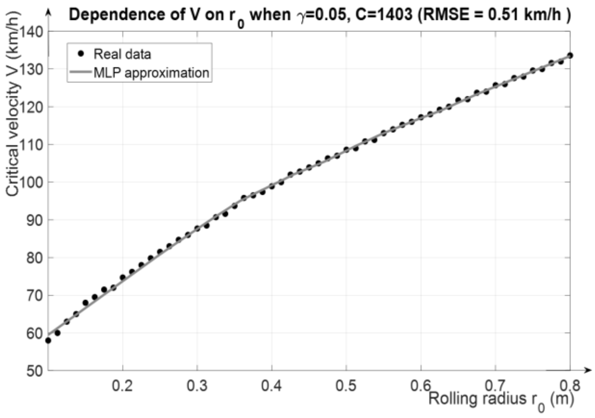

Fig. 7. Comparison of critical velocity vs. wheelset conicity with other parameters held constant

As shown in Figure 7, MLP approximates the critical velocity with an error of $RMSE = 0.51\ km/h$ indicating a very high level of accuracy.

Discussion of results

The comparative analysis of the three machine learning models – Multilayer Perceptron (MLP), Gaussian Process Regression (GPR), and Support Vector Regression (SVR) – reveals distinct performance patterns depending on dataset size and underlying parameter dependencies.

All models were trained using a limited number of parameter combinations. As shown in Figures 1–3, on the small dataset both MLP and GPR successfully captured the nonlinear relationship governing critical speed, achieving high fidelity with respect to reference values derived from the high-fidelity curvilinear motion model [8]. Notably, GPR exhibited marginally superior accuracy ($R^2 = 0.9987$) attributable to its Bayesian nature and inherent ability to infer smooth functional forms from sparse data. In contrast, SVR (Figure 3) failed to generalize: its predictions showed significant deviation, particularly at the extremes of the conicity range, yielding $R^2 = 0.9721$ indicating poor adaptability to the underlying dynamics.

When scaling to the large dataset, the behavior diverged markedly (Figures 4–6). MLP's performance improved: R^2 value rose to 0.9997 confirming its strong learning capacity and scalability with increasing data volume. This aligns with the known strengths of deep feedforward networks in approximating high-dimensional nonlinear mappings when sufficiently trained. Conversely, GPR's accuracy degraded. SVR remained consistently unreliable across both dataset sizes. In summary, the results demonstrate that: MLP offers the best trade-off between accuracy, robustness, and scalability; GPR is suitable for small-data prototyping but becomes inefficient and less accurate as data grows; SVR is unsuitable for this specific regression task under the tested configurations.

Conclusions

Three supervised regression methods – Multilayer Perceptron (MLP), Gaussian Process Regression (GPR), and Support Vector Regression (SVR) – were comparatively evaluated for their ability to approximate the critical velocity of railway rolling stock under strongly nonlinear parameter dependencies and limited data availability; Appropriate metrics were selected to evaluate models performance, and a comparative assessment of their effectiveness and accuracy was conducted. The results show that both MLP and GPR achieve high prediction accuracy on both small and large datasets, with MLP demonstrating superior scalability and improved accuracy as the dataset size increases, whereas GPR performance degrades under the same conditions. In contrast, SVR exhibits insufficient prediction accuracy across all dataset sizes.

A further experiment involved isolating the dependence of critical speed on wheelset conicity γ under fixed r_0 and C confirms not only global approximation capability but also local sensitivity preservation – a critical requirement for real-time safety systems such as the Driver Decision Support System (DDSS)

Consequently, the MLP model is selected for further implementation as a core component of the critical velocity estimation module within the Driver Decision Support System (DDSS) and as part of the train curvilinear motion model.

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Received (Надійшла) 26.09.2025
Accepted for publication (Прийнята до друку) 14.10.2025

ВІДОМОСТІ ПРО АВТОРІВ/ ABOUT THE AUTHORS

Євтушенко Олександр Сергійович – аспірант кафедри комп'ютерної інженерії та програмування, Національний технічний університет "Харківський політехнічний інститут", Харків, Україна;
Oleksandr Yevtushenko – PhD student, Department of Computer Engineering and Programming, National Technical University 'Kharkiv Polytechnic Institute', Kharkiv, Ukraine;
e-mail: Oleksandr.Yevtushenko@cs.khpi.edu.ua; ORCID Author ID: https://orcid.org/0009-0000-7798-8545.

Заковоротний Олександр Юрійович – доктор технічних наук, професор, завідувач кафедри комп'ютерної інженерії та програмування, Національний технічний університет "Харківський політехнічний інститут", Харків, Україна;
Oleksandr Zakovorotnyi - Doctor of Technical Sciences, Professor, Head of the Department of Computer Engineering and Programming, National Technical University 'Kharkiv Polytechnic Institute', Kharkiv, Ukraine;
e-mail: Oleksandr.Zakovorotnyi@khpi.edu.ua; ORCID Author ID: https://orcid.org/0000-0003-4415-838X;
Scopus Author ID: https://www.scopus.com/authid/detail.uri?authorId=57201613700.

ДОСЛІДЖЕННЯ ЕФЕКТИВНОСТІ МЕТОДІВ ОБЧИСЛЕННЯ КРИТИЧНОЇ ШВИДКОСТІ РУХУ ПОЇЗДА

О. С. Євтушенко, О. Ю. Заковоротний

Анотація. Актуальність. Визначення критичної швидкості залізничного рухомого складу є ключовим етапом при проектуванні та експлуатації сучасних поїздів, оскільки перевищення цієї швидкості призводить до стійких поперечних коливань, зростання бічних сил у контакті колеса з рейкою і, як наслідок, збільшення ризику сходу рухомого складу з колії. У зв'язку з погіршенням стану рухомих складів і шляхового господарства, стає необхідним забезпечення надійного та ефективного прогнозування критичної швидкості на основі сучасних обчислювальних методів. **Предметом дослідження** є ефективність методів машинного навчання для апроксимації критичної швидкості в умовах нелінійних залежностей та обмеженого обсягу даних. **Метою статті є** відбір найбільш ефективного методу обчислення критичної швидкості поїзда для подальшого впровадження в систему підтримки прийняття рішень машиніста (СППРМ) та як компонента моделі криволінійного руху поїзда**. Були отримані наступні результати**: моделі MLP (Multilayer Perceptron) та GPR (Gaussian Process Regression) продемонстрували високу точність прогнозування як на малому, так і на великому обсязі даних, причому MLP показав кращу масштабованість при зростанні обсягу навчальних даних, ніж GPR. **Висновки.** На основі проведеного порівняльного аналізу доцільно використовувати MLP як основну модель для визначення критичної швидкості в СППРМ та діагностики залізничного транспорту.

Ключові слова: критична швидкість; коливання рухомого складу; багатошаровий перцептрон; гаусівська регресія; опорні вектори; апроксимація; машинне навчання; СППРМ; криволінійний рух.